Concatenation of Space-Time Block Codes and Turbo Product Codes over Rayleigh Flat Fading Channels

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***Abstract*—Space-Time Block Codes(STBCs) are designed to obtain the maximum possible diversity gain. However, STBCs are not designed to achieve an additional coding gain [1,2]. Therefore, STBCs need to be concatenated with an outer code which provides a significant coding gain. Turbo Product Codes (TPCs) are a kind of high-efficient coding scheme in high code rate, which more approach the Shannon limit than do turbo convolutional code(TCC)[3]. In this paper, we consider the concatenation of TPCs with STBCs to improve the reliability of the wireless communication systems. We combined turbo product code$(64, 57, 4)^2$ as an outer code. STBCs-TPCs system with Multiple-transmit and single-receive antenna(MISO) are investigated for Rayleigh flat fading channels. Simulation results show that for one system with total spectral efficiency of 0.793 bits/s/Hz, Bit Error Rate (BER) is 0 at E_b/N_0 of 7dB using 4 Transmit antennas(4Tx) or at E_b/N_0 of 12dB using 3Tx or at E_b/N_0 of 21dB for 2Tx and that for another system with total spectral efficiency of 1.19 bits/s/Hz , BER is 0 at E_b/N_0 of 8dB using 4Tx or at E_b/N_0 of 15dB using 3Tx.**

Keywords-multiple antennas; space-time block codes; turbo product codes; MISO; Rayleigh flat fading channels

I. INTRODUCTION

In future 4G mobile communication systems, high data rates need to be reliably transmitted over time-varying band limited channels. The wireless channel mainly suffers from time-varying fading due to multipath propagation and destructive superposition of signals received over different paths, which makes it hard for the receiver to reliably decide in favour of the transmitted signal unless some less attenuated replicas of the signal are provided to the receiver.

Transmitting the replicas of the signal is called diversity. There are several ways—time diversity, frequency diversity, and antenna diversity, by which we can provide the receiver with independently fading replicas of the same information-bearing signal. A widely applied technique to reduce some detrimental effects of multipath fading is antenna diversity. Usually, multiple antennas are used at the receiver with some kind of combining of the received signals, e.g., maximum ratio combining. However,it is hard to efficiently use receive antenna diversity at the remote units since they should remain relatively simple, inexpensive and small.Hence, receive diversity has been nearly exclusively used at base station[4].

STBCs were introduced to provide transmit diversity in wireless fading channels using multiple transmit antennas. An STBC can be represented by a matrix whose row is the number of time slots for transmitting one block of symbols and column is the number of transmit antennas [5]. However, STBCs are not designed to achieve an additional coding gain, unless concatenated with an outer code. In digital mobile communication systems, powerful channel coding is paramount to combat the effects of fading, interference and noise to obtain sufficient reception quality.

In the past few years, several concatenation schemes that combine STBCs with turbo codes, multilevel codes or LDPC codes have been proposed to improve Bit Error Rate(BER) performance of an STBC receiver in [6,7,8]. Turbo product codes introduced in [9] can achieve remarkable error performance at a low Signal-to-Noise Ratio(SNR), which exhibit no an error floor at low BERs that have been attributed to other codes. TPCs are also capable of providing high coding gain, even with high code rates [10].

In this paper, we introduce a new scheme for wireless communication systems, which combines the Turbo product codes coded modulation with space-time block codes. Encoding and Decoding of turbo product codes are briefly reviewed in section Ⅱ. In section Ⅲ and section Ⅳ, encoder and decoder designs of STBCs system based on TPCs are introduced. The simulation results and conclusion are detailed in section Ⅴand Ⅵ respectively.

II. ENCODING AND DECODING OF TURBO PRODUCT CODES

A. *Encoding Flow Chart of Turbo Product Codes*

The Encoding flow chart of TPCs is shown in Figure 1.TPCs are built from a multidimensional array of (N,K) block codes,where (N,K) block codes are named as subcodes. According to the subcode class of constituent codes, there are Reed Solomon product codes, BCH product codes, extended Hamming product codes with an additional parity check bit and single parity product codes. Different code rates are achieved by using various length codes in two or three dimensions.

An example of a two-dimensional(2-D) TPC encoding scheme is a row extended Hamming(eH) code C_1 (N_1,K_1,d_1) ,

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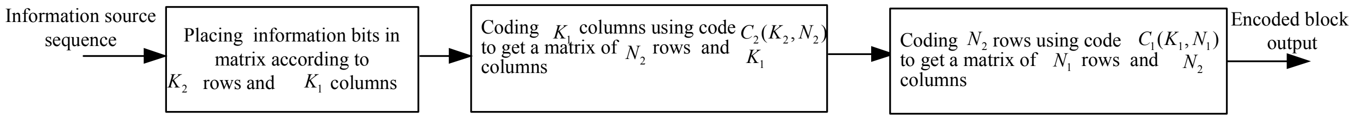

Figure 1. Encoding block diagram of TPC

multiplied by a column extended Hamming (eH) code $C_2(N_2,K_2,d_2)$, i.e.eH(64, 57, 4)×eH(64, 57, 4), where 64, 57, 4 stand for the length of a codeword, the length of information bits and the minimum Hamming distance respectively. The code rate of this TPC encoder is $R_{tpc}=(K_1\times K_2)/(N_1\times N_2)=(57\times 57)/(64\times 64)=0.793$ and its minimum Hamming distance is $D_{min}=d_1\times d_2=4\times 4=16$. Such a TPC code can correct up to $s=\lfloor (D_{\min}-1)/2\rfloor=\lfloor (16-1)/2\rfloor=\lfloor 7.5\rfloor=7$ error bits, where $\lfloor X\rfloor$ is the largest integer no greater than the real number X. Due to distance structure of TPCs, the error correction capability of TPCs is strengthened significantly.

B. Decoding of Turbo Product Codes

The decoding algorithm for TPCs applies an iterative decoding method to a product array of extended Hamming or single parity check codes. The conventional serial iterative decoding method[9] for turbo product codes array is to decode each row using soft decision correlation decoding. The output of the row decoding is then combined to the original data and input to a decoder for each column using soft decision correlation decoding. The output of the column decoding is input back to the row decoding. This process continues until the decoder settles on a valid transmitted code array or until the maximum number of iterations is reached. The whole procedure [11] is shown in Figure 2.

As an example of a 2-D TPC, for the first iteration, we initialize $L(d)=0$ (assume equally likely information, d denote $k_1\times k_2$ information bits). The row decoding of the TPC starts using the corresponding $L_c(x)$ for the information part and for check part of row. The row extrinsic information $L_e^-(\hat{d})$ on the information bits d is from

$$L_e^-(\hat{d}) = L^-(\hat{d}) - L_c(x) \tag{1}$$

This independent estimate on d is now used as the a priori value for column decoding to obtain

$$L_e^{|}(\hat{d}) = L^{|}(\hat{d}) - [L_c(x) + L_e^-(\hat{d})] \tag{2}$$

This column extrinsic information will be used as a new priori value in the subsequent decoding of code C_1 in the next iteration step. Note that for the first row and the first column iteration the L-values are statistically independent, but since later on they will use the same information indirectly, they will become more and more correlated and finally the improvement through the iterations will be marginal. Of course, for the final decision (or soft output) after the last column iteration we combine the last two extrinsic pieces of information with the received values to obtain

$$L(\hat{d}) = L_c(x) + L_e^-(\hat{d}) + L_e^{|}(\hat{d}) \tag{3}$$

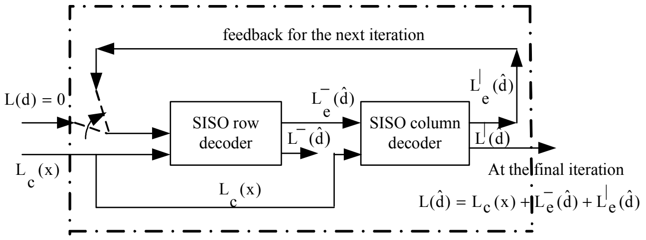

Figure 2. Serial iterative decoding procedure of 2-D TPCs with two "Soft Input Soft Output" decoders

which, using (2),is identical to $L^{|}(\hat{d})$.

III. ENCODER DESIGN OF STBC SYSTEM BASED ON TPCS

It is essential that the STBCs-TPCs system is the concatenation of STBCs and TPCs. The proposed system is shown in Figure 3. A parallel stream of bit $\boldsymbol{X}=\{x_1,......,x_n\}$ is sent to the TPC encoder. The encoded symbol is denoted by $\boldsymbol{Y}=\{y_1,......,y_m\}$ ($m>n$). Following the block interleaver and modulator, the modulated symbol is denoted by $\boldsymbol{S}=\{s_1,......,s_k\}$. If the STBC encoder inputs four symbols represented by $\boldsymbol{S}=\{s_1,s_2,s_3,s_4\}$, the transmitted signals in three transmit antennas are a generalized complex orthogonal matrix given by

$$\boldsymbol{G_3}=\begin{pmatrix} s_1 & -s_2 & -s_3 & -s_4 & s_1^* & -s_2^* & -s_3^* & -s_4^* \\ s_2 & s_1 & s_4 & -s_3 & s_2^* & s_1^* & s_4^* & -s_3^* \\ s_3 & -s_4 & s_1 & s_2 & s_3^* & -s_4^* & s_1^* & s_2^* \end{pmatrix} \tag{4}$$

where $\boldsymbol{G_3}$ is composed of linear combinations of constellation symbols s_1,s_2,s_3,s_4 and their conjugates. The number of row denotes the number of transmit antennas and the number of column denotes the number of time slots for transmitting an STBC code word. During the first time slot, the three symbols in the first column $[s_1,s_2,s_3]$ of $\boldsymbol{G_3}$ are transmitted simultaneously from three transmit antennas; during the second time slot, the three symbols in the second column $[-s_2,s_1,-s_4]$ of $\boldsymbol{G_3}$ are transmitted. The rest may be deduced by analogy and the code rate of the STBC is $R_{STBC}=1/2$. Each of the three transmitted signals(or symbols) first goes through a Rayleigh flat fading channel separately. They are then combined together and AWGN is added to form the output signal.

Similarly, if the input to the STBC encoder is three symbols represented by $\boldsymbol{S}=\{s_1,s_2,s_3\}$, the transmitted signal in three transmit antennas can be given in transmission matrix $\boldsymbol{G_3'}$

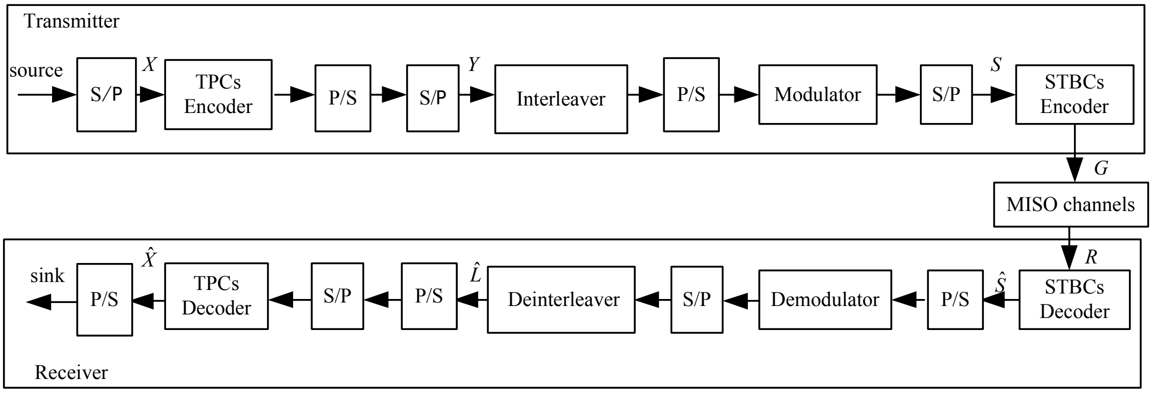

Figure 3. Block diagram of STBCs-TPCs system over MISO Rayleigh flat fading channels

$$\boldsymbol{G}_3' = \begin{pmatrix} s_1 & -s_2^* & s_3^*/\sqrt{2} & s_3^*/\sqrt{2} \\ s_2 & s_1^* & s_3^*/\sqrt{2} & -s_3^*/\sqrt{2} \\ s_3/\sqrt{2} & s_3/\sqrt{2} & \dfrac{(-s_1 - s_1^* + s_2 - s_2^*)}{2} & \dfrac{(s_2 + s_2^* + s_1 - s_1^*)}{2} \end{pmatrix} \tag{5}$$

Then $R_{STBC} = 3/4$ is obtained. If the average total power transmitted on three antennas is P_s at each transmission interval(or each time slot), we deduce that each entry of the input to STBC has power $E_s = 4P_s/9$.

IV. DECODER DESIGN OF STBC SYSTEM BASED ON TPCS

Referring to Figure 3, we consider the wireless communication system with N transmit antennas and one receive antenna. At time t , the received signal r_t is given by

$$r_t = \sum_{n=1}^{N} h_n g_{t,n} + \eta_t \tag{6}$$

where h_n is channel gain; $g_{t,n}$ is the codeword transmitted at time t , from transmit antenna n ; the noise samples η_t are modeled as independent samples of a zero mean complex Gaussian random variables, the real and imaginary part of η_t have equal variance $N_0/2$. In our proposed system , we define the Signal-to-Noise Ratio(SNR) as P_s/N_0 , where P_s is the total power at receiver, N_0 is the noise spectral density. Assuming perfect channel state information can be achieved and the channel is quasi-static , the receiver computes the following decision metric

$$\sum_{t=1}^{T} \left| r_t - \sum_{n=1}^{N} h_n g_{t,n} \right|^2 \tag{7}$$

over all codewords $g_{t,n}$ and decides in favour of the codeword that minimizes this sum[2].

For (4),the output of the linear combiner at the receiver is given by

$$\hat{s}_1 = r_1 h_1^* + r_2 h_2^* + r_3 h_3^* + r_5^* h_1 + r_6^* h_2 + r_7^* h_3 \tag{8}$$

$$\hat{s}_2 = r_1 h_2^* - r_2 h_1^* + r_4 h_3^* + r_5^* h_2 - r_6^* h_1 + r_8^* h_3 \tag{9}$$

$$\hat{s}_3 = r_1 h_3^* - r_3 h_1^* - r_4 h_2^* + r_5^* h_3 - r_7^* h_1 - r_8^* h_2 \tag{10}$$

$$\hat{s}_4 = -r_2 h_3^* + r_3 h_2^* - r_4 h_1^* - r_6^* h_3 + r_7^* h_2 - r_8^* h_1 \tag{11}$$

V. SIMULATION RESULTS

Simulation parameters are set up as shown in Table 1. BPSK modulation is used for the R_{STBC} =1 STBC, QPSK for the R_{STBC} = 1/2 STBC and the R_{STBC} = 3/4 STBC to obtain the same spectral efficiency (i.e. two cases: 1 bit/s/Hz and 1.5 bits/s/Hz) for fair comparisons.

While the information bit stream is encoded by the TPCs encoder, is then modulated by BPSK or QPSK constellations and finally transmitted from the STBCs encoder, the

TABLE I. SIMULATION PARAMETERS

Parameters	Specification
Number of antenna Branches	2, 3, 4
TPCs Structure	(64,57,4)×(64,57,4)
STBCs	R_{STBC} = 1 studied by Alamouti ; R_{STBC} = 1/2 with 4 input symbols and 8 time slots; R_{STBC} = 3/4 with 3 input symbols and 4 time slots
Symbol Constellation	BPSK for Alamouti code; QPSK for other STBCs
Iteration Number of TPCs decoder	16
Block Interleaver Length	4096
The average total power transmitted on all the antennas at each transmission interval	P_s =1
Channel Model	a Rayleigh flat fading channel, one separate path each antenna
Carrier Frequency	1. 850GHz
Sampling Frequency	1 .0MHz
Default Doppler Frequency	17.12962963Hz
The speed of the mobile terminal	10 km/h

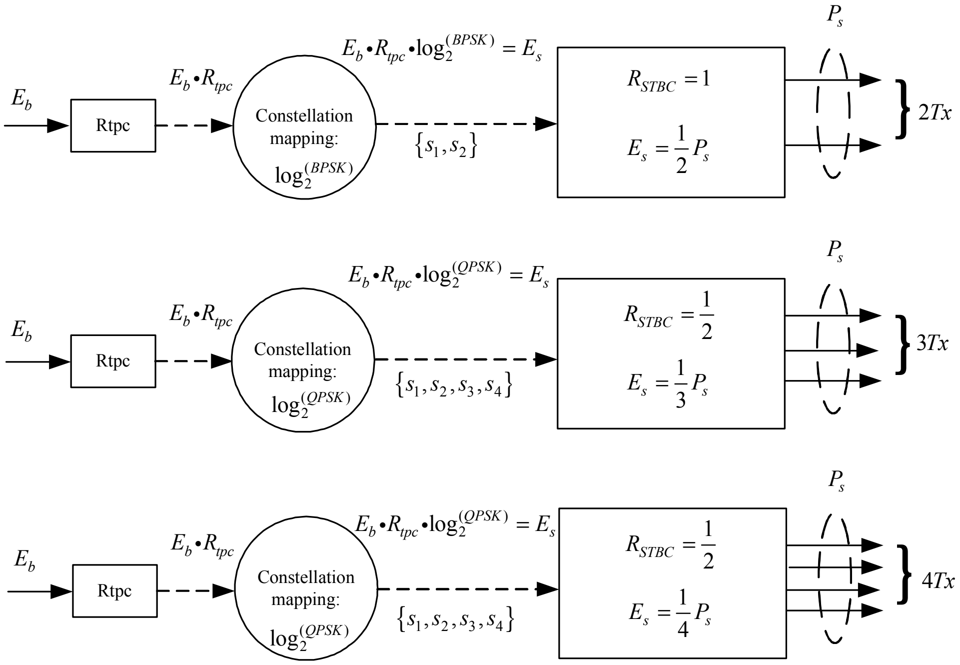

Figure 4. The relationships between E_b , E_s and P_s for STBCs- TPCs transmitter using 2Tx,3Tx or 4Tx with total spectral efficiency of 0.793 bits/s/Hz

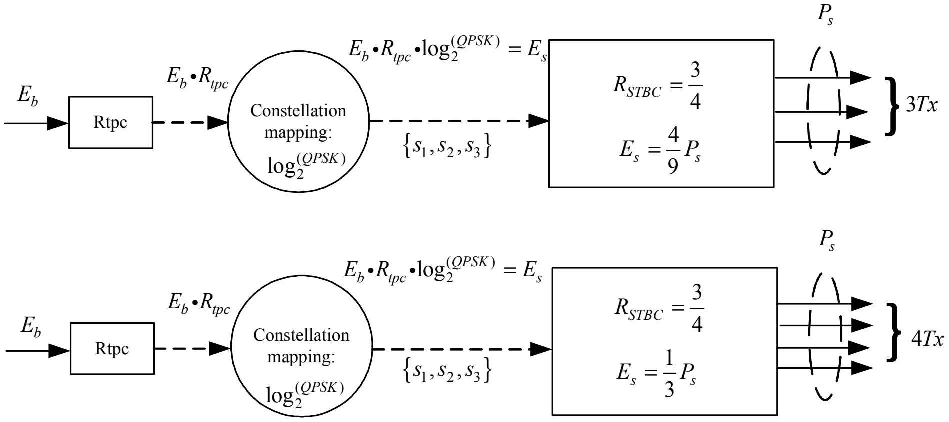

Figure 5. The relationships between E_b , E_s and P_s for STBCs- TPCs transmitter using 3Tx or 4Tx with total spectral efficiency of 1.19 bits/s/Hz

relationships between E_b , E_s and P_s are illustrated in Figure 4 and 5, where E_b , E_s , and P_s denotes each data bit energy ,each symbol constellation energy, and the average total power transmitted on all the antennas at each time slot respectively.

For STBCs-TPCs system with 2Tx-1Rx, 3Tx-1Rx or 4Tx-1Rx, referring to Figure 4 and 5, E_b / N_0 is calculated as follows, in decibel notation,

$$E_b / N_0 = SNR - 10 \bullet \lg(R_{tpc} \bullet \alpha \bullet \log_2^{(MPSK)}) \qquad (12)$$

where $\alpha = P_s / E_s$, E_s is the average energy of a MPSK constellation point.

The Simulations were performed on a Signal Processing Worksystem (SPW) software. BER performance of STBCs-TPCs system using 2Tx-1Rx, 3Tx-1Rx or 4Tx-1Rx with total spectral efficiency of 0.793 bits/s/Hz is depicted in Figure 6 and that of STBCs-TPCs system using 3Tx-1Rx or 4Tx-1Rx with total spectral efficiency of 1.19 bits/s/Hz is plotted in Figure 7. The Solid vertical lines stand for BER of 0 under the point. It can be seen that the clear diversity gain increases at each additional transmit antenna.

Figure 6 shows 4Tx-1Rx system performance exceeds 3Tx-1Rx and 2Tx-1Rx system performance about 5 dB and 12 dB respectively at a BER of 10^{-4}. From Figure 7, it is observed

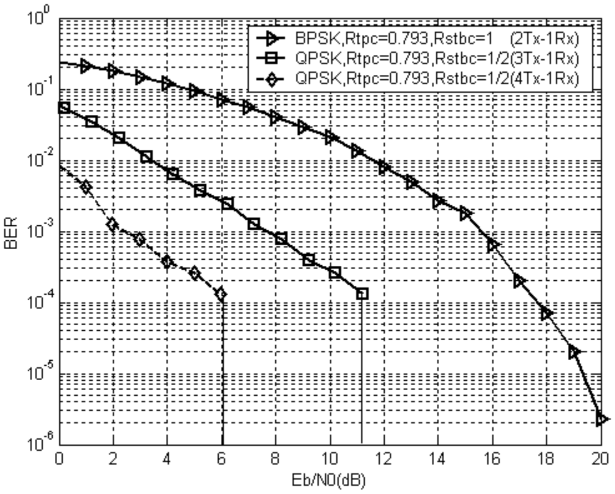

Figure 6. BER performance of STBCs-TPCs system using 2Tx-1Rx, 3Tx-1Rx or 4Tx-1Rx with total spectral efficiency of 0.793 bits/s/Hz

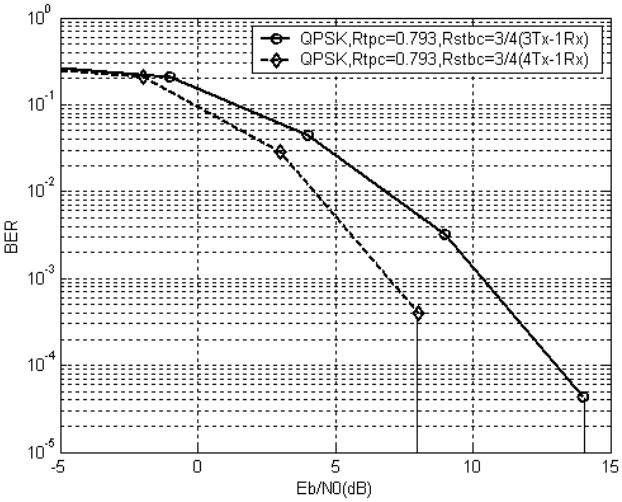

Figure 7. BER performance of STBCs-TPCs system using 3Tx-1Rx or 4Tx-1Rx with total spectral efficiency of 1.19 bits/s/Hz

that 4Tx-1Rx system performs approximately 3dB better than 3Tx-1Rx system at BER of 10^{-4}.

VI. CONCLUSION

In this paper, BER performance of STBCs-TPCs system with multiple-transmit antennas and a single-receive antenna over Rayleigh flat fading channels is compared. This comparison is useful for design and implementation of multiple-antenna wireless systems. Concatenating STBCs to TPCs achieves the diversity gain from the transmit diversity and the coding gain from the concatenated TPCs. In the next step, STBCs-TPCs techniques based on MIMO-OFDM will be further investigated.

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